# **Some Activities for Teaching Fractions**

This is an excerpt from Changing the Way We Teach Math by Kate Nonesuch. The full manual is available on her website at

There are two sets of activities here: in the first, students use manipulatives to demonstrate for themselves some concepts and operations with fractions, and in the second, students work in groups at the board. The ideas in one set practice and reinforce the ideas in the other set; there is a lot of practice and review involved in each, and a measured but quite quick movement towards more complex mathematical ideas.

I have used both these sets of activities successfully for many years, but would refer the reader to the discussions of student resistance and of group work in Chapters 2 and 5 of <u>Changing the Way We Teach Math</u> to ensure a firm underpinning for these activities. See also "<u>Math Strategies in Context</u>"

## **Fractions Demonstrations**

The activities on the following pages require students to use math tools (manipulatives) to demonstrate their understanding of fractions and to show their answers are correct. For example, take this question:

Circula dha ann allan fua adiana	1	1	
Circle the smaller fraction:	$\overline{2}$	12	✓

The student's job is to set out the manipulatives that represent one half and one twelfth, showing that the twelfth is smaller. The teacher will come to see the result, have a short conversation, and will initial the blank space to show that the demonstration has been completed.

Some students will find the answers to the questions first by using math notation and algorithms they know; when they go to demonstrate that their answer is correct, they may find that, indeed, it is not correct, and may then be in a position to rethink their algorithm, or to come to a clearer understanding of what the abstraction of the algorithm conceals. The manipulatives give them a chance to correct their mistake before it gets marked wrong by the instructor. Correcting their own errors and showing they are right increases confidence and understanding.

Other students, who do not know the algorithms, or who have less faith in their memory of them, will go directly to finding the answers simply by manipulatives. As they learn the algorithms from their texts or in class, they will begin to combine the two methods, with the learning from the experience with manipulatives helping them remember and understand the algorithms.

The checkmark symbol  $\checkmark$  \_\_\_\_\_ next to a question reminds the student that the answer must be demonstrated with the tools. As students work on the activities, the instructor can circulate and sign off demonstrations as students get them ready. Encourage students to compare answers with one another, and, in the case of a disagreement, use the tools to figure out who is right.

#### Signing off the demonstrations

 $\checkmark$  \_\_\_\_\_ Wherever this symbol appears, students should prove their answer is correct by setting up a demonstration with the math tools. Your job is to look at their demonstrations, ask questions to clarify or extend their understanding, and sign or initial on the line when you are satisfied. Encourage students to line up several demonstrations as they wait for you to circulate to their desks, rather than setting up one and waiting. Students can work individually or in pairs.

#### What should you look for?

Each question is shown below, with some suggestions for dealing with the demonstrations students offer.

#### 1. Write these fractions in order, smallest to largest.

This question is an example of the usefulness of manipulatives—if the student writes the answer incorrectly before doing the proof, the manipulatives will show him what the correct answer is, and he will change his written answer to line up with the proof before you get there. You can come along and sign off on the demonstration without having to mark anything wrong, and can then engage in a conversation about how counter-intuitive the structure is, and have some conversation about the meaning, i.e., that the bottom number tells how many pieces the whole thing is divided into, and if you cut more pieces, each piece will have to be smaller.

#### 2. Circle the smaller fraction.

Ask, "How can you tell by the way the fractions are written which one is smaller?" Look for an answer like "A bigger bottom number means the whole is cut into more pieces, so each piece is smaller.

#### 3. Circle the smaller fraction.

Ask, "How can you tell by the way the fractions are written which one is smaller?" An answer like "When the bottom numbers are the same, you know the pieces are the same size, so you can just look at the top number to see how many pieces there are," means that the student is beginning to see the necessity of a common denominator when comparing fractions.

#### 4. Write a fraction that equals one whole.

Ask, "How can you tell by the way the fraction is written that it is equal to 1?" You are asking the student to articulate the fact that a fraction with the same top and bottom number always equals 1. (Except 0.) The manipulatives give the student a chance to find this out for himself, rather than hearing it from you.

#### 5. Write a fraction that equals one half.

Ask, "How do you know your answer is right?" and expect a variety of answers on the theme of "The top number is half of the bottom number."

#### 6. Write a fraction that is the same size as the one given.

Showing these equivalents for frequently used fractions helps to build up a visual memory that students can recall when they use standard methods from the texts to write equivalent fractions.

#### 7-8. Put these fractions in the right space on the chart below.

These questions help students get a sense of the size of different fractions. Ask, "How do you know this is in the right place?"

#### 9. Write these fractions in order from smallest to biggest. 7/8, 1/12, 5/4, 3/3.

By using the skills from the previous two examples, students can do this question without having to change everything to a common denominator. 1/12 is less than half, 7/8 is more than half, 3/3 is equal to 1 and 5/4 is larger than 1. Nice to show that an understanding of fractions can help students do mentally what would take a lot of work and the possibility of many errors to do by the standard method of changing to a common denominator.

#### 11. Circle the question if the answer will be more than 1.

This skill can help with estimating answers to addition and subtraction questions before you do them, or with making a quick check to see if an answer is reasonable after doing an adding or subtracting question. Ask, "How do you know your answer is right?"

#### 12-19.

Students may use these demonstrations as a way of understanding whatever algorithm they use to do questions of these types.

# Fraction Demonstrations

A check mark tells you to use the tools to show your answer. Show your work with the tools to the teacher, who will sign beside the check mark.

1. Write these fractions in order, smallest to largest.										
$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{1}$	$\frac{1}{\pi}$	$\frac{1}{1}$	✓		
2	12	8	3	6	4	5	10			
When the bottom number is <b>smaller</b> , the fraction is										
When t	he botton	n numbe	er is <b>big</b>	ger, the	fraction	is				
2 Cir	le the sn	naller fr	ection							
<b>2.</b> CII	cie the sh		action.			1	1			
$\frac{1}{2}$	$\frac{1}{12}$	✔_				6	3	✓		
2	12									
						1	1			
$\frac{1}{4}$	$\frac{1}{12}$	✓_				5	$\overline{8}$	✓		
·	12									
When t	he botton	n numbe	er is		, th	e fracti	on is <b>smalle</b>	r.		
When t	When the bottom number is, the fraction is <b>bigger</b> .									
2 Cincle the smaller fraction										
J. CIT		uanci II	acuvii.			2	1			
$\frac{1}{12}$	$\frac{3}{12}$	✓_				10	$\overline{10}$	✓		
14	14									



### 6. Write a fraction that is the same size as the one given.

$\frac{1}{2} = \frac{1}{10}$	v	۱	_	$\frac{1}{4} = \frac{1}{12}$	2	✓ _			$\frac{1}{3} = \frac{1}{6}$	6	✔_		-
$\frac{1}{2} = \frac{1}{12}$	v	۱	_	$\frac{1}{4} = \frac{1}{8}$		✔_			$\frac{1}{3} = \frac{1}{3}$	12	✓_		
$\frac{1}{2} = \frac{1}{4}$	v	I	_	$\frac{1}{4} = \frac{1}{16}$	-	✓ _			$\frac{2}{3} = \frac{1}{6}$	6	✓_		
$\frac{1}{2} = \frac{1}{8}$	v	I	_	$\frac{3}{4} = \frac{1}{12}$	-	✓ _			$\frac{2}{3} = \frac{1}{1}$	12	✓_		
$\frac{1}{2} = \frac{1}{6}$	v	·	_	$\frac{3}{4} = \frac{1}{8}$		✓ _			$\frac{1}{6} = \frac{1}{1}$	12	✓_		
$\frac{1}{2} = \frac{1}{16}$	v	•	_	$\frac{3}{4} = \frac{1}{16}$	5	✓ _			$\frac{2}{5} = \frac{1}{10}$	$\overline{0}$	✓		
$\frac{1}{5} = \frac{1}{10}$	v	•	_	$\frac{5}{6} = \frac{1}{12}$	2	✓ _			$\frac{4}{5} = \frac{1}{10}$	$\overline{0}$	✓		
7. Put th	iese frac	ctions i	in the rig	ght space	on th	ne chart	below.						
$\frac{4}{8}$	$\frac{11}{12}$	$\frac{3}{5}$	$\frac{1}{3}$	$\frac{5}{6}$		$\frac{1}{4}$	$\frac{4}{5}$	$\frac{5}{10}$		$\frac{9}{10}$		$\frac{1}{8}$	$\frac{2}{3}$
Less the	an 1/2		Equal t	o 1/2		More	than 1/2						]
L													_

	×

8. Put these fractions in the right space in the chart below.

5	12	5	5	5	4	10	3	11	1	4
8	12	5	$\overline{3}$	$\overline{6}$	4	$\overline{10}$	$\overline{5}$	$\overline{10}$	$\overline{8}$	$\overline{3}$

Less	s than 1		Equal to	1	More than 1		
						·	
9. W	rite these	fraction	ns in order	from smalle	est to biggest.		
$\frac{7}{8}$	$\frac{1}{12}$	$\frac{5}{4}$	$\frac{3}{3}$				✓
$\frac{7}{6}$	$\frac{3}{6}$	$\frac{3}{4}$	$\frac{3}{3}$				<b>ب</b>

10. Reduce to lowest terms. This means to write an equal fraction with a bottom number as low as possible. For example:  $\frac{2}{4} = \frac{1}{2}$ 



$\frac{4}{6} =$	✓	$\frac{8}{10} =$	✓
$\frac{4}{8} =$	✓	$\frac{2}{8} =$	✓
$\frac{3}{12} =$	✔	$\frac{5}{10} =$	✓
$\frac{6}{8} =$	✓	$\frac{2}{12} =$	✓
$\frac{6}{12} =$	✔	$\frac{4}{12} =$	✓
$\frac{8}{16} =$	✓	$\frac{4}{10} =$	✓
$\frac{8}{12} =$	✓	$\frac{9}{12} =$	✓
$\frac{4}{16} =$	✔	$\frac{8}{16} =$	✓
$\frac{12}{16} =$	✔	$\frac{14}{16} =$	✓



11. Circle the question if the answer will be more than 1. You do not have to give the answer.

$\frac{1}{2}$	+	$\frac{11}{12}$	✓	$\frac{1}{2}$ + $\frac{1}{6}$	✓
$\frac{1}{2}$	+	$\frac{3}{10}$	✓	$\frac{9}{10}$ + $\frac{3}{4}$	✓
$\frac{1}{2}$	+	$\frac{3}{8}$	✓	$\frac{3}{4} + \frac{3}{4}$	✓
$\frac{1}{2}$	+	$\frac{1}{24}$		$\frac{5}{8}$ + $\frac{75}{100}$	
$\frac{1}{6}$	+	$\frac{1}{3}$	✓	$\frac{1}{2}$ + $\frac{7}{8}$	✓

12. Add. Reduce the answer if you can.



### 13. Subtract. Reduce the answer if you can.

### 14. Add. Reduce the answer if you can.



1		3		1	+	3	
$\frac{1}{3}$	+	$\frac{3}{4}$	✓	16		4	✓

15. Subtract. Reduce the answer if you can.



### 17. Subtract. Reduce the answer if you can.



				4		7	
3	_	5		$\overline{5}$	_	$\overline{10}$	
4		8	•				•

18. Multiply. Remember, you can read "of" when you see "x."



### **19.** Divide. Divide the first number into pieces the size of the second number. How many pieces are there?

1	÷	$\frac{1}{3}$	✓	$\frac{1}{2}$ ÷	$\frac{1}{4}$	✓
2	÷	$\frac{1}{5}$	✔	$\frac{6}{10}$ ÷	$\frac{1}{5}$	✓
$\frac{2}{3}$	÷	$\frac{1}{3}$	✔	$\frac{1}{2}$ ÷	$\frac{1}{8}$	<b>پ</b>
$\frac{5}{8}$	÷	$\frac{1}{8}$	✓	$\frac{1}{2}$ ÷	$\frac{1}{10}$	✓

# Fractions at the Board

#### Time: 15-20 minutes a day

The following series of activities with fractions gives students practice with fractions, allows you to check their understanding and their ability to manipulate fractions, and gives students a chance to articulate what they know. It also shows the value of review and over-learning; students will notice as it gets easy to answer questions similar to the ones that were difficult the week before. Students should find most of these questions easy, because they are doing many examples of similar questions until it seems easy. If every question is hard, they will be reluctant to go to the board. Every day only a few questions should present some challenge. You can tailor the questions to suit your students, asking more questions similar to ones they are learning, or skipping some questions that they are finding too easy or too difficult at the moment. For some students you can repeat one day's activities for several days, using different numbers.

When students can give a correct response quickly and easily, that is the time to ask them to explain their thinking, since usually the skill of talking about math lags behind the skill of doing math. This means that students of different abilities can be working on the same content area with the same questions: for some at a lower level, just doing the work is the job at hand; for others at a higher skill level, articulating the process is the job that requires work and keeps the interest high.

When students are familiar with the process, ask a student to "be the teacher" and read the instructions for other students to follow. The challenge of checking that student responses are correct, while running the group process and noticing patterns, will be interesting math experience for some students whose skills in doing the math are more advanced than most of the group.

### Process

Each day, ask students to go the board to read and write some fractions. Encourage them to make themselves at home, to look around at other students' work, to stand beside someone who is good at math, to get a long piece of chalk, and so on. They should do whatever they need to be comfortable at the board. Especially, make sure there is an eraser between every pair of students. If a student has to ask for an eraser, he calls attention to his error. We want him to take a risk, so make sure he can quickly and quietly erase his answer if it is wrong.

Every day, the work follows the same pattern:

- 1. As you call out the first instruction, students will write the fraction(s).
- 2. Quickly look around and make sure that every response is correct, helping individuals as necessary.
- 3. Ask students to look around to see if their answer is the same as everyone else's. Different answers may be correct, and provide an opportunity to talk about the process.
- 4. Repeat for each of the bulleted instructions on the day.
- 5. Thank students for their participation and their excellent work, and invite them to sit down.

### Day 1

Think about all the people in the room. What fraction are men? Write that fraction. Write the fraction of people who are women. What fraction are teachers? What fraction are students?

- Think about the people at the front board. What fraction are men? Write that fraction. Write the fraction of people at the front board who are women.
- Think about the people at the side board. What fraction are men? Write that fraction. Write the fraction of people at the side board who are women.
- Suppose there were twelve people in the room and half were women. Write a fraction with 12 on the bottom that shows the fraction of women in that room. Correct. 6/12 is equivalent to 1/2.
- Suppose there were 100 people in the room and half were men. Write a fraction with 100 on the bottom that shows the fraction of men in that room. Correct. 50/100 is equivalent to 1/2.
- Write five fractions equivalent to 1/2.
- I'll give you three fractions, 1/2, 1/10 and 1/4. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- Thanks. You may sit down.

- Think about all the people in the room. What fraction are wearing glasses? Write that fraction. Write the fraction of people who are not wearing glasses.
- Think about the people at the front board. What fraction are wearing glasses? Write that fraction.
- Think about the people at the side board. What fraction are wearing glasses? Write that fraction.
- Repeat until everyone can write a fraction easily, asking for the fraction of people wearing shorts, wearing skirts, wearing black, or wearing watches, etc. Ask the question of the whole room, and then of one or more smaller groups of students (for example, front board and side board), so that the denominators are not the same all the time. Sometimes include yourself, sometimes not, for example, "What fraction of the people in the room are wearing glasses?"
- Suppose there were six pieces of pizza left after the party, and half had mushrooms on them. Write a fraction with 6 on the bottom that shows the fraction of pieces that had mushrooms. Correct. 3/6 is equivalent to 1/2.
- Write five other fractions equivalent to 1/2.
- I'll give you three fractions, 1/6, 1/2 and 1/100. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- I'll give you three fractions, 5/8, 1/8 and 3/8. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- Thanks. You may sit down.

- Think about the people in this room. What fraction of the women are wearing black shoes today?
- What fraction of the men are wearing black shoes today?
- What fraction of the students are wearing black shoes today?
- Repeat until everyone can write a fraction easily, asking for the fraction wearing shorts, wearing skirts, wearing black, or wearing watches, etc. Ask the question of the whole room, and then of one or more smaller groups of students, so that the denominators are not the same all the time.
- Write five fractions equivalent to 1/2.
- I'll give you five fractions, 1/6, 1/2, 1/4, 1/12 and 1/100. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- I'll give you four fractions, 4/7, 3/7, 1/7 and 7/7. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- (Use this question after you have taught a process for generating equivalent fractions.) Write a fraction equivalent to 1/2 with a bottom number of 4. (Write the question up in the usual way so that people can see it. 1/2 = /4). Continue with a few more: 1/4 = /8; 3/4 = /8; 1/3 = /6; 2/3 = /6.
- Thanks, you can sit down.

- Ask them to write several fractions describing people in the room, e.g., What fraction of the men have their hair tied back? What fraction of students have their hair tied back? What fraction of women?
- Write five fractions equivalent to 1/2.
- I'll give you five fractions, 1/10, 1/2, 1/4, 1/5 and 1/20. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- I'll give you four fractions, 4/10, 3/10, 5/10 and 9/10. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- Write some equivalent fractions: a fraction equivalent to 1/2 with a bottom number of 4. (Write the question up in the usual way so that people can see it. 1/2 = /4). Continue with a few more: 1/4 = /12; 3/4 = /16; 1/3 = /9; 2/3 = /12.
- I eat 3/4 of a chocolate bar. What fraction is left?
- The kids eat 5/8 of the pizza. What fraction is left?
- The team plays the first quarter of the basketball game. How much of the game is left?
- I do 1/3 of my homework. What fraction is left?
- Thanks, you can sit down.

- Draw three columns at the board. At the top of one write "less than 1/2," at the top of the next, write "= 1/2," and at the top of the last column write "more than 1/2." I'll give you some fractions. You write them in the correct column: 1/4, 7/8; 5/10, 3/4, 5/6, 1/12, 3/100.
- Write five fractions equivalent to 1.
- I'll give you five fractions, 1/3, 1/10, 1/5, 1/9, and 1/2. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- I'll give you four fractions, 89/100, 39/100, 51/100 and 9/100. Write them in order from smallest to largest. If you like, draw some diagrams.
- Write some equivalent fractions, 1/4 = /12; 1/3 = /12; 2/3 = /12; 3/4 = /12, 5/6 = /12.
- I watch 3/4 hours of a one-hour program. What fraction is left?
- I eat 1/2 of an apple. What fraction is left?
- The kids use up 4/5 of the toothpaste. What fraction is left?
- I watch the first half of the soccer game. How much of the game is left?
- I have to read 10 pages for homework. I read 9 pages. What fraction of my homework is left?
- Reduce these fractions to lowest terms: 2/4, 4/8, 6/12, 8/16, 12/24.

- Draw three columns at the board. At the top of one write "less than 1/2;" at the top of the next, write "= 1/2," and at the top of the last column write "more than 1/2." I'll give you some fractions. You write them in the correct column: 5/10, 4/8, 1/100, 6/12, 9/14, 10/20, 3/6.
- Write five fractions equivalent to 1.
- I'll give you five fractions, 1/5, 1/14, 1/7, 1/4, and 1/2. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- I'll give you four fractions, 5/8, 1/8, 3/8, and 8/8. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- Write some equivalent fractions: 1/4 = /16; 1/3 = /15; 2/3 = /15; 3/5 = /20; 5/6 = /24.
- I watch 1/4 hour of a one-hour program. What fraction of the program is left?
- We eat 2/3 of a pizza. What fraction of the pizza is left?
- I drive 9/10 of the way to Nanaimo. What fraction of the trip is left?
- I have five hours of homework to do. I work for three hours. What fraction of the work is left?

- The Yankees played 5 innings against the Red Sox. What fraction of the game is left?
- Reduce these fractions to lowest terms: 4/10, 2/12, 4/14, 6/9, 12/16, 7/21.

- Draw three columns at the board. At the top of one write "less than 1/2," at the top of the next, write "= 1/2," and at the top of the last column write "more than 1/2." I'll give you some fractions. You write them in the correct column: 4/5, 1/8, 91/100, 8/15, 7/14, 5/10, 6/6.
- Write five fractions equivalent to 1.
- Write five fractions larger than 1.
- I'll give you three fractions, 1/6, 9/10 and 2/4. Write them in order from smallest to largest. Think about whether they are bigger or smaller than 1/2, or equivalent to 1/2.
- I'll give you five fractions, 5/9, 1/9, 7/9, 2/9 and 9/9. Write them in order from smallest to largest.
- Write some equivalent fractions: 1/4 = /20; 2/3 = /18; 3/5 = /15; 3/8 = /16; 5/9 = /18.
- I drink 1/2 of my cup of coffee. What fraction is left?
- We walk 3/4 of a block. What fraction is left?
- There were 10 questions on the test. I got 7 right. What fraction did I get wrong?
- The teams played one period of hockey. What fraction of the game is left?
- Reduce these fractions to lowest terms: 4/20, 2/28, 5/15, 6/15, 5/35, 7/35.
- Add: 3/4 + 1/4; 1/8 + 3/8; 4/9 + 3/9.
- Subtract: 1 1/2; 1 3/4; 1 2/3.

- Draw three columns at the board. At the top of one write "less than 1/2," at the top of the next, write "= 1/2," and at the top of the last column write "more than 1/2." I'll give you some fractions. You write them in the correct column: 5/8, 1/21, 91/100, 7/13, 6/12, 9/18, 3/3.
- Write five fractions equivalent to 1.
- Write five fractions larger than 1.
- Write five fractions smaller than 1.
- Draw three columns. At the top of one write "less than 1," at the top of the next, write "= 1," and at the top of the last column write "more than 1." I'll give you some fractions. You write them in the correct column: 5/8, 4/4, 9/10, 3/2, 7/7, 4/1, 6/6.
- I'll give you three fractions, 1/6, 10/10 and 6/4. Write them in order from smallest to largest. Think about whether they are bigger or smaller than 1, or equivalent to 1.

- I'll give you five fractions, 5/3, 1/3, 2/3, 3/3 and 10/3. Write them in order from smallest to largest.
- Write some equivalent fractions: 1/4 = /16; 2/3 = /21; 3/5 = /40; 3/8 = /32; 5/9 = /45.
- The third quarter of the football game is over. What fraction of the game is left to play?
- Four team mates run a relay race. What fraction of the race does each person run?
- Reduce these fractions to lowest terms: 4/10, 7/21, 8/16, 6/18, 5/25, 7/56.
- Add: 1/2 + 1/2; 1/2 + 1/4; 1/2 + 1/8.
- Subtract: 1/2 1/2; 1/2 1/4; 1/2 1/8.

#### **Following days**

Many days of board work follow this pattern—adding and subtracting, multiplying and dividing fractions, continuing to build up a sense in students of the relative size of fractions and mixed numbers.